

1. Define what is meant by ( 2 pts each )
  - a. A mapping  $\alpha : A \rightarrow B$  is one-to one .
  - b The index of a subgp  $H$  in a gp  $G : [ G : H ]$  .
  - c. The kernel of a homom  $\theta : G_1 \rightarrow G_2$  .
2. ( 5 pts ) Let  $\alpha = (153)(26)$  &  $\beta = (1342)$  be in  $S_6$  .  
In cycle notation right – hand mult. Find :
  - i)  $\beta^{-1}\alpha$  , ii)  $\alpha^2$       iii)  $(\alpha\beta)^{-1}$       iv) Is  $\alpha$  even or odd ? Why ?
3. ( 4 pts ) a. Write the lattice diagram for  $S_3$  .  
b. Show that  $S_3$  is not comm.
4. ( 4 pts ) a. Show that  $H \cap K \leq G$  , for any two subgps  $H$  and  $K$  of a gp  $G$  .  
b. Is  $H \cup K$  always a subgp of  $G$  also ? Show your claim .
5. (4pts) a. Find the g.c.d. (3960 , 2880 )  
b. Find the l.c.m. (3960 , 2880 )
6. ( 4 pts ) a. Write down the elements of  $Z_2 \times S_3$  .  
b. Show that  $A \times B$  is abelian iff  $A$  &  $B$  are abelian .
7. ( 4 pts ) a. State Lagrange's Thm .  
b. Show that if  $G$  is a gp  $\exists |G| = p$  , a prime number , then  $G$  is cyclic .
8. ( 4 pts ) a. Find  $[ Z_{18} : \langle 3 \rangle ]$  .  
b. Let  $G$  be an abelian gp ,  $H \leq G$  . Show that  $H \triangleleft G$  .
9. ( 5 pts ) Let  $\beta : (R, +) \rightarrow (R^+, \times) \exists$   
$$x \mapsto e^x \quad , \forall x \in R$$
  
Show that  $\beta$  is an isom . of gps. .

END