

Student Name: _____ Student No.: _____

Part I (25 points): Choose the correct answer.

1. Simplify $||-2| - 4|$.

- (a) 6 (b) 2 (c) -6 (d) -2

2. Simplify $\left(\frac{8a^{-4}b^3}{27a^2b^{-3}}\right)^{\frac{1}{3}}$.

- (a)
- $\frac{3a^2}{2b^2}$
- (b)
- $\frac{2}{3a^2}$
- (c)
- $\frac{3a^2b^2}{2}$
- (d)
- $\frac{2b^2}{3a^2}$

3. Factor completely $wy - wz + 2xy - 2xz$.

- (a)
- $(w + 2x)(y + z)$
- (b)
- $(w + 2x)(y - z)$
-
- (c)
- $(w - 2x)(y + z)$
- (d)
- $(w - 2x)(y - z)$

4. If $f(x) = \begin{cases} 5 & \text{if } x \leq -1 \\ 2x + 3 & \text{if } -1 < x < 1 \\ 6x^2 - 4 & \text{if } x \geq 1 \end{cases}$. Find $2f(1) - f(-1)$.

- (a) -1 (b) 3 (c) 5 (d) 1

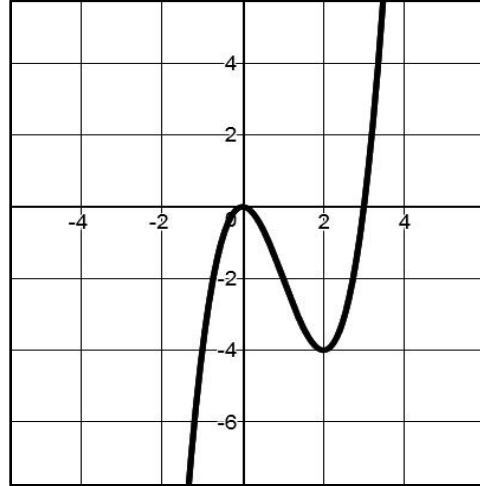
5. Find the center and the radius of the circle whose equation is

$x^2 + y^2 - 4x + 6y + 9 = 0$.

- (a)
- $(-2, 3)$
- and
- $r = 3$
- (b)
- $(-2, 3)$
- and
- $r = 2$
-
- (c)
- $(2, -3)$
- and
- $r = 2$
- (d)
- $(2, -3)$
- and
- $r = 3$

6. Given the graph of a function. Find the interval in which it is decreasing.

- (a) $(-\infty, 0) \cup (2, \infty)$
- (b) $(0, 2)$
- (c) $(-4, 0)$
- (d) $(-\infty, -4) \cup (0, \infty)$



7.
$$\frac{8x^4 + 6x^2 - 3x + 1}{2x^2 - x + 2} =$$

- (a) $7x - 1 + \frac{4x^2 - 2x}{2x^2 - x + 2}$
- (b) $4x^2 - 2x + \frac{7x - 1}{2x^2 - x + 2}$
- (c) $4x^2 + 2x + \frac{-7x + 1}{2x^2 - x + 2}$
- (d) $-7x + 1 + \frac{4x^2 + 2x}{2x^2 - x + 2}$

8. Find the solution of the equation $3^{x-1} = 5$.

- (a) $1 + \log_5 3$
- (b) $\frac{1}{3} \log_3 5$
- (c) $\log_3 6$
- (d) $1 + \log_3 5$

9. Use the laws of logarithm to re-write the expression as a single logarithm.

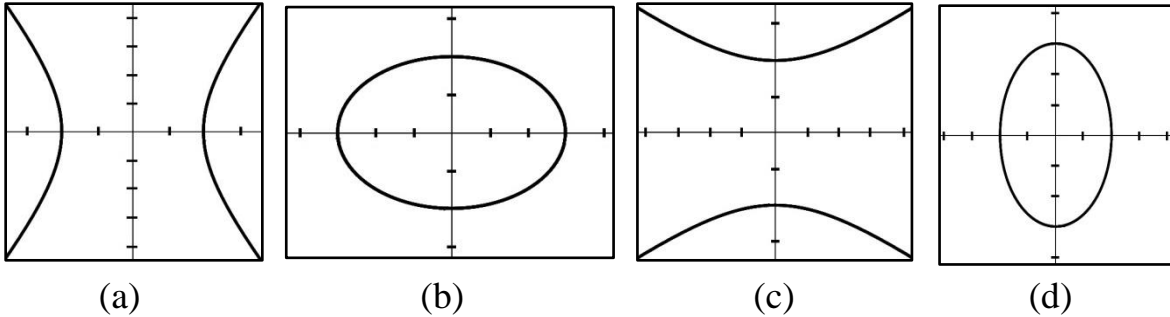
$$\frac{1}{2} \ln y + \ln(y^2 - 1) - \ln(y + 1)$$

- (a) $\ln[\sqrt{y}(y - 1)]$
- (b) $\ln\left[\frac{1}{2}y(y - 1)\right]$
- (c) $\ln\left[\frac{\sqrt{y}}{(y-1)}\right]$
- (d) $\ln\left[\frac{1}{2}y + y^2 - y\right]$

10. What are the vertices of the hyperbola $5x^2 - 9y^2 = 45$?

- (a) $(3, \sqrt{5})$ and $(-3, -\sqrt{5})$ (b) $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$
 (c) $(3, 0)$ and $(-3, 0)$ (d) $(0, 3)$ and $(0, -3)$

11. Which of the following is the graph of the equation $9x^2 + 4y^2 = 36$?



12. Convert 210° to radians measure ?

- (a) $\frac{5\pi}{3}$ (b) $\frac{7\pi}{6}$ (c) $\frac{4\pi}{3}$ (d) $\frac{3\pi}{2}$

13. Simplify $\frac{\sin(\frac{\pi}{2} - x) \sec x}{\tan x}$.

- (a) $\cos x - \sin x$ (b) $\csc x$ (c) 1 (d) $\cot x$

14. Find the exact value of $\sin(41^\circ) \cos(19^\circ) + \cos(41^\circ) \sin(19^\circ)$.

- (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) 1

15. Given $\cos(\theta) = \frac{-3}{5}$, and θ is in quadrant III. Find the value of $\tan \theta$.

- (a) $\frac{-4}{5}$ (b) $\frac{-5}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

Part II (25 points): Solve each of the following questions completely.

1. (5 points) Find the solution of the logarithmic equation

$$\log_3(x + 1) + \log_3(x) = \log_3(3x + 8).$$

Ans:

$$\log_3(x^2 + x) = \log_3(3x + 8)$$

$$\Rightarrow x^2 + x = 3x + 8$$

$$\Rightarrow x^2 + x - 3x - 8 = 0$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4 \quad \text{or} \quad x = -2$$

Check:

at $x = 4$

$$\log_3(4 + 1) + \log_3(4) = \log_3(3 \times 4 + 8)$$

at $x = -2$

$$\log_3(-2 + 1) + \log_3(-2) = \log_3(3 \times -2 + 8)$$

Rejected

$$\Rightarrow S = \{4\}$$

2. (5 points) Find the solution set of the trigonometric equation in the interval $[0, 2\pi)$.

$$2 \tan \theta \sin \theta - \tan \theta = 0.$$

Ans:

$$\tan \theta (2 \sin \theta - 1) = 0$$

Either

$$\tan \theta = 0$$

$$\theta = 0$$

$$\theta = \pi$$

Or

$$2 \sin \theta - 1 = 0$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

Reference angle is : $\hat{\theta} = \frac{\pi}{6} = 30^\circ$

1st quadrant $\theta = \frac{\pi}{6} = 30^\circ$

2nd quadrant $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

or $\theta = 180^\circ - 30^\circ = 150^\circ$

$$\Rightarrow S = \left\{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\right\} \quad \text{Or} \quad S = \{0, 180^\circ, 30^\circ, 150^\circ\}$$

3. (5 points) Find the solution of the inequality $\frac{3x}{x+1} > x$.

$$\frac{3x}{x+1} - x > 0 \Rightarrow \frac{3x - x(x+1)}{x+1} > 0 \Rightarrow \frac{3x - x^2 - x}{x+1} > 0 \Rightarrow$$

$$\frac{2x - x^2}{x+1} > 0 \Rightarrow \frac{x(2-x)}{x+1} > 0$$

4. $-\infty \quad -1 \quad 0 \quad 2 \quad \infty$

x	-	-	+	+
$2-x$	+	+	+	-
$x+1$	-	+	+	+
$f(x)$	+	-	+	-

$$\Rightarrow S = (-\infty, -1) \cup (0, 2)$$

4. (2.5 + 2.5 = 5 points) Given $f(x) = \frac{x}{x+4}$ and $g(x) = \frac{1}{x}$. Find:

- $(f \circ g)(x)$.
- The domain of $(f + g)(x)$.

Ans:

$$i. (f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}}{\frac{1}{x} + 4} = \frac{\frac{1}{x}}{\frac{1+4x}{x}} = \frac{\frac{1}{x}}{\frac{1+4x}{x}} =$$

$$\frac{1}{1+4x}$$

ii. The domain of $(f + g)(x)$:

$$D_f = \mathbb{R}/\{-4\}$$

$$D_g = \mathbb{R}/\{0\}$$

$$\Rightarrow D_{f+g} = D_f \cap D_g = \mathbb{R}/\{-4, 0\}$$

5. (5 points) Let $P(x) = x^3 - 2x + 1$.
Find all real zeros of $P(x) = 0$.

Ans:

$$P(1) = 0$$

\Rightarrow 1 is a zero of $P(x)$

1	1	0	-2	1
		1	1	-1
	1	1	-1	0

$$Q(x) = x^2 + x - 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{5}}{2}$$

The zeros are $x = 1$, $x = \frac{-1}{2} \pm \frac{\sqrt{5}}{2}$

Answer Key:

1	B	9	A
2	D	10	C
3	B	11	D
4	A	12	B
5	C	13	D
6	B	14	A
7	C	15	D
8	D		