Kuwait University	Math. 240	June 11, 2013
Dept. of Mathematics	First Midterm Exam	Duration 90 Minutes

Calculators of any kind are not allowed.

All communication devices must be switched off.

Answer the following questions.

Each question is worth 20 Points.

1. Solve the differential equation

$$\left[y^2 - x^2 \sec\left(\frac{y}{x}\right)\right] dx - xydy = 0$$

2. Solve the differential equation

$$\frac{dy}{dx} = \frac{e^{y^2}}{y\left(1 - 2xe^{y^2}\right)}$$

3. Solve the initial value problem

$$xy' = 2y\left(x^4 y - 1\right);\ y(-1) = 1$$

4. Solve the differential equation

$$\left(y^2 - 2y\sin x\right)\cos x dx + \left(y^2 + \sin^2 x - 1\right) dy = 0$$

5. Find the orthogonal trajectories of the family of curves

$$x^2 + 3y^2 = cy$$

1. $y = xv \Longrightarrow dy = xdv + vdx \Longrightarrow \dfrac{dx}{x} + v\cos v dv = 0$

$$\ln|x| + v\sin v + \cos v = c \implies \boxed{\ln|x| + \left(\frac{y}{x}\right)\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right) = c.}$$

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2. $e^{y^2}dx + \left(2xye^{y^2} - y\right)dy = 0 \Longrightarrow d\left(xe^{y^2} - \frac{1}{2}y^2\right) = 0$

$$\boxed{2xe^{y^2} - y^2 = c.}$$

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3. $y' + \left(\dfrac{2}{x}\right)y = 2x^3y^2 \ \ (BE) \Longrightarrow z' - \left(\dfrac{2}{x}\right)z = -2x^3 \ \ (LE)\,; \ z = y^{-1}$

$$\mu(x) = \exp\left(-\int\frac{2dx}{x}\right) = x^{-2} \implies x^{-2}z = -2\int xdx + c = -x^2 + c$$

$x^{-2}y^{-1} = -x^2 + c$. For $x = -1$ and $y = 1 \Longrightarrow c = 2 \implies \boxed{x^2y\left(2 - x^2\right) = 1.}$

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4. $t = \sin x \implies \left(y^2 - 2yt\right)dt + \left(y^2 + t^2 - 1\right)dy = 0$

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial t}\right) = \frac{2y - 4t}{y\left(y - 2t\right)} = \frac{2}{y} \implies \mu(y) = \exp\left(-\int\frac{2dy}{y}\right) = \frac{1}{y^2}$$

$\left(1 - \dfrac{2t}{y}\right)dt + \left(1 + \dfrac{t^2}{y^2} - \dfrac{1}{y^2}\right)dy = 0$ is Exact Eq. $\Longrightarrow t - \dfrac{t^2}{y} + y + \dfrac{1}{y} = c$

$$\boxed{(y - \sin x)\sin x + y^2 + 1 = cy.}$$

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5. $x^2 + 3y^2 = cy \ \& \ 2x + 6yy' = cy' \implies 2xy + \left(3y^2 - x^2\right)y' = 0 \ \ (DE)$

$\left(x^2 - 3y^2\right)dx + 2xydy = 0 \ \ (DE)_\perp$. This Eq. can be solved as (HE), (BE) or by finding I.F as follows

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-6y - 2y}{2xy} = -\frac{4}{x} \implies \mu(x) = \exp\left(-4\int\frac{dx}{x}\right) = \frac{1}{x^4}$$

$\left(\dfrac{1}{x^2} - \dfrac{3y^2}{x^4}\right)dx + \dfrac{2y}{x^3}dy = 0$ is Exact Eq. $\Longrightarrow -\dfrac{1}{x} + \dfrac{y^2}{x^3} = k$

$$\boxed{y^2 - x^2 = kx^3.}$$
