Dept. of Mathematics	MATH 240 - ODE	Kuwait University
Summer Semester 2011	Final Test	Tuesday, 09 August 2011

Duration: 120 min. Total marks: 40.

Instructions: Answer the following questions & show all work in order to receive full credit.

1. [5 pts. each] Find the general solution for the following differential equations:

(a) $[2\cos(2x+y)-x^2]dx+[\cos(2x+y)+e^y]dy=0, \qquad y(\pi)=0.$

(b) $\left[x^2+(\tan^{-1}y)^2\right]dx+\frac{x^2}{1+y^2}dy=0, \qquad y(1)=0.$

2. [5 pts.] Let $y_1 = x$ be one solution of Legendre's equation

$$(1-x^2)y''-2xy'+2y=0, \qquad -1<x<1$$

Find the second linearly independent solution y_2.

3. [5 pts.] Solve using Laplace transform method: $y''(t)+4y(t)=-e; \quad y(0)=3,$ and use the constraint $y\left(\frac{\pi}{4}\right)=0$, to find the value of $y'(0)=b$.

4. [5 pts.] Use the step function $\alpha(t-c)$ to find the Laplace transform of $H(t)$, where

$$H(t)=\begin{cases} t^2-2, & 0\le t<1 \\ 2-t, & 1\le t<3 \\ 7, & t>3 \end{cases}$$

5. [3+4 pts.] If $f(s)=L\{F(t)\}$ & $a>0$. Use the definition of Laplace transform to:

(a) show that $f(as)=\frac{1}{a}L\left\{F\left(\frac{t}{a}\right)\right\}$.

(b) Using part (a), prove that if $\phi(t)=\int_0^t \sin\left(\frac{t-\beta}{a}\right)\cos\left(\frac{\beta}{a}\right)d\beta$ then

$$L\{\phi(t)\}=\frac{a^3 s}{((as)^2+1)^2}.$$

6. [4+4 pts.]

(a) Find a formula for the coefficients of the power series solutions near the origin for the following homogeneous equation

$$(1+x^2)y''-4xy'+6y=0, \qquad y(0)=\frac{1}{6},\ y'(0)=4.$$

(b) Show that the coefficients in the power series solutions near the origin of the following nonhomogeneous equation

$$(1+x^2)y'' - 4xy' + 6y = e^{2x}$$

are recursively defined by the relations:

$$a_n = \frac{\left[\dfrac{2^{n-2}}{(n-2)!} - (n-4)(n-5)a_{n-2}\right]}{n(n-1)}, \quad n \geq 2.$$
