Kuwait University	Math. 240	April 12, 2010
Dept. of Math. & Comp. Sc.	First Midterm	Duration 90 Minutes

Calculators and mobile phones are not allowed.
Answer all questions. Each question is worth 5 points.

1. **(4 Points)** Classify the following differential equations as the type of. (**Justify your answer**)
   - **a.** $ydx + 3x - \ln y)dy = 0$
   - **b.** $(3x^2y^2 - 1)dx + 2x^3ydy = 0$
   - **c.** $y' = \sin(x + y) + \sin(x - y)$
   - **d.** $ydx + (x - x^2y^2 \ln y)dy = 0$
2. **(5 Points)** Solve the following equation
$$xy' + y\ln x = y\ln y.$$
3. **(4 Points)** Solve the differential equation
$$(xy + y + x + 1)dx - (x^2 + 1)dy = 0.$$
4. **(5 Points)** Solve the initial value problem
$$\cosh x dx + (\sinh x \cot y + 2y \csc y)dy = 0;\ y(0) = \frac{\pi}{2}.$$
5. **(2 Points)** Let y be a non-zero solution of
$$y' + y^n = 0$$
For What values of n, cy_1 is a solution of the differential equation if $c \neq 0$.
6. **(5 Points)** Find the orthogonal trajectories of the family of curves
$$x^2 - x(y - 3) + y^2 = c^2.$$

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1. (a) Linear in x (b) Exact (c) Separation of variables (d) Bernoulli in x

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2. The equation is homogeneous. Put $y = xv \Rightarrow y' = xv' + v \Rightarrow \frac{dx}{x} = \frac{dv}{v(\ln v - 1)}$

$\ln|\ln v - 1| = \ln|x| + c \Rightarrow$ $\boxed{y = xe^{1+kx};\ k = e^c.}$

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3. The equation is a separation of variables type. It can be written as

$$(y+1)(x+1)dx - (x^2+1)dy = 0$$

which becomes

$$\left(\frac{1}{x^2+1} + \frac{x}{x^2+1}\right)dx = \frac{1}{y+1}dy \Rightarrow \boxed{2\tan^{-1}x + \ln(x^2+1)(y+1)^2 = c.}$$

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4. Put $u = \sinh x$. Then the equation becomes $du + (u\cot y + 2y\csc y)dy = 0$.
This is a linear equation in u,

$$\frac{du}{dy} + (\cot y)u = -2y\csc y$$

The integrating factor is $\sin y$, and therefore the solution is given by

$$(\sin y)u = \int -2y\csc y\sin y dy = -y^2$$

The general solution is then given by $\boxed{\sin y\sinh x = -y^2 + c.}$

and the particular solution is then given by $\boxed{\sin y\sinh x = -y^2 + \frac{\pi^2}{4}.}$

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5. After substitution of cy_1 into the equation, it becomes
$cy_1' + c^n y_1^n = c(y_1' + c^{n-1}y_1^n) = 0.$ If $c \neq 0$, then $n-1 = 0$, $i.e.$ $\boxed{n = 1.}$

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6. The differential equation associated to this family is given by

$$(2x - y + 3)dx + (2y - x)dy = 0.$$

So, the differential equation associated to the family of orthogonal trajectories is given by

$$(2y - x)dx - (2x - y + 3)dy = 0.$$

This is an equation with linear constant coefficients. Let $x = u - 2$ and $y = v - 1$. Thenthe equation becomes an homogeneous

$$\frac{dv}{du} = \frac{2v - u}{2u - v}$$

Taking $w = \frac{v}{u}$, we obtain

$$\frac{du}{u} = \frac{2 - w}{w^2 - 1}dw.$$

After integration we get $\ln|u| = \frac{1}{2}\ln|w - 1| - \frac{3}{2}\ln|w + 1| + c.$ So,the family of orthogonal trajectories has the equation

$$\boxed{\ln|x + 2| = \frac{1}{2}\ln\left|\frac{y-x-1}{x+2}\right| - \frac{3}{2}\ln\left|\frac{y+x+3}{x+2}\right| + c.}$$