Kuwait University	Math. 240	November 10, 2009
Dept. of Math. & Comp. Sc.	First Midterm	Duration 90 Minute

Calculators and mobile phones are not allowed.
Answer all questions. Each question is worth 5 points.

1. Solve the following equation
$$\frac{dy}{dx} = \frac{(1+2x)\,e^{y}}{y+x^2y}.$$

2. Solve the differential equation
$$2x^3yy' - x^4 = y^4.$$

3. Find the general solution of the equation
$$(y\ln y + ye^{x})\,dx + (x - y\cos y)\,dy = 0.$$

4. Solve the initial value problem
$$\left(1+x^2\right)y' = 2xy - 3y^2\sin x;\ \ y(0) = -1.$$

5. Find the orthogonal trajectories of the family of curves
$$y\left(x^2+c\right) = 1.$$

1. $(1+2x)\, e^y dx - (y + x^2 y)\, dy = 0 \Longrightarrow \dfrac{1+2x}{1+x^2} dx - ye^{-y} dy = 0$

$$\boxed{\tan^{-1} x + \ln(1+x^2) + (1+y)\, e^{-y} = c.}$$

---

2. $y = tx \Longrightarrow y' = xt' + t \Longrightarrow \dfrac{dx}{x} = \dfrac{2tdt}{t^4 - 2t^2 + 1} = \dfrac{2tdt}{(t^2-1)^2}$

$$\ln|x| + \frac{1}{t^2-1} = c \Longrightarrow \ln|x| + \frac{1}{t^2-1} = c$$

$$\boxed{\ln|x| + \frac{x^2}{y^2 - x^2} = c.}$$

---

3. $\dfrac{1}{M}\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right) = \dfrac{\ln y + e^x}{y(\ln y + e^x)} = \dfrac{1}{y} \Longrightarrow \mu(y) = \exp\left(-\int \frac{dy}{y}\right) = \dfrac{1}{y}$

$(\ln y + e^x)\, dx + \left(\dfrac{x}{y} - \cos y\right) dy = 0$ is exact $\Longrightarrow \boxed{x \ln y + e^x - \sin y = c.}$

---

4. $y' - \dfrac{2x}{1+x^2} y = \left(-\dfrac{3\sin x}{1+x^2}\right) y^2 \;\; (BE) \Longrightarrow z' + \left(\dfrac{2x}{1+x^2}\right) z = \dfrac{3\sin x}{1+x^2} \quad (LE);\;\; z = y^{-1}$

$(1+x^2)\, y^{-1} = 3\int \sin x dx + c = -3\cos x + c \Longrightarrow \boxed{(1+x^2)\, y^{-1} = -3\cos x + c.}$

Or. $(1+x^2)\, dy - 2xydx + 3y^2 \sin x dx = 0 \Longrightarrow d\left(\dfrac{1+x^2}{y}\right) + d(3\cos x) = 0$

$(1+x^2)\, y^{-1} + 3\cos x = c$

$x = 0$ and $y = -1 \Longrightarrow c = 2 \;\; \Longrightarrow \boxed{(1+x^2)\, y^{-1} = 2 - 3\cos x.}$

---

5. $y(x^2 + c) = 1 \Longrightarrow x^2 y' + 2xy + cy' = 0 \Longrightarrow y' + 2xy^2 = 0 \;\; (DE)$

$dx - 2xy^2 dy = 0 \;\; (DE)_\perp \Longrightarrow \dfrac{1}{x} dx - 2y^2 dy = 0$

$$\boxed{2y^3 - 3\ln|x| = k.}$$
